

Finite Element Modeling of Liquefaction-induced Deformations on Square Foundations

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Abstract: The presence of problematic soils in construction zones often leads to significant issues such as swelling, settlement, liquefaction, and others, which can cause serious damage to structures. This research focuses on liquefaction, specifically by conducting a numerical study on the engineering and mechanical behavior of these soils under loading by shallow square foundations. The numerical study, performed using the finite element method (FEM) with PLAXIS software, shows that the behavior of problematic soils, depending on their nature, typically leads to a reduction in soil strength and stability, resulting in foundation instability and failure. Based on the results from the numerical modeling conducted using PLAXIS, the geotechnical behavior of the main problematic soils has been determined. This research also proposes improvement and stabilization approaches for shallow foundations, particularly under shear stress-strain conditions and displacement measurements around the foundation. The findings suggest that the numerical method and PLAXIS software are effective in evaluating deformations and modeling the behavior of problematic soils.

Keywords: Square foundations, Problematic soil, Geotechnical engineering, Finite element method, PLAXIS.

I. INTRODUCTION

The study of problematic soils covers a broad spectrum of geotechnical applications in soil mechanics and design. These soils, due to their unique characteristics, can significantly affect geotechnical structures either built with or within them. They tend to compromise stability and contribute to structural failures. Problematic soils are generally categorized into four primary groups and two secondary groups. The primary categories include liquefiable soils, expansive soils, dispersive soils, and collapsible soils, while the secondary categories consist of artificial soils and dissolvable soils (Yuan & Cao, 2011). In Iran, these primary soil types are frequently encountered at various construction sites and have historically caused

considerable damage. This research delves into an empirical analysis and a deeper understanding of these soils' properties.

One of the natural phenomena that can cause significant damage is soil liquefaction. This phenomenon is considered a primary cause of failure in saturated, silty sand deposits. Although it wasn't fully recognized until 1953, it was first identified by Kubo in that year, bringing liquefaction to the forefront as a critical topic in geotechnical engineering (Ansari & Mansouri-Kia, 2016). A major turning point came with the 1964 Niigata earthquake in Japan, which had a magnitude of 7.5 on the Richter scale. Niigata, a city that had been rebuilt with modern infrastructure after a devastating fire in 1955, suffered unprecedented destruction due to this earthquake. The 1964 Niigata earthquake is often cited as a symbolic event, being one of the first in the world to completely devastate an advanced city's lifelines. The primary cause of the damage was soil liquefaction, which consequently drew significant attention from the geotechnical community. Since the Niigata earthquake, seismic geotechnical engineers have worked extensively to standardize the mechanisms of liquefaction and develop guidelines based on field and laboratory data (Zhang et al., 2021).

Another notable example is the liquefaction that occurred in Alaska, USA, following the 1964 earthquake, which led to the destruction of roads and buildings. Similarly, the 1989 Loma Prieta earthquake caused sand boils and structural damage due to liquefaction (Verudgo, 2005). In Iran, liquefaction was also a key factor in the widespread damage during the June 21, 1990, earthquake in Astaneh-ye Ashrafiyeh and surrounding villages. The Manjil-Rudbar earthquake of the same year severely impacted the region due to liquefaction (Asgari & Qasemi, 2009). While there have been many other instances of liquefaction in Iran, the damage was not as extensive as in the cases, yet it still caused significant harm to infrastructure and urban services.

Typically, when an earthquake occurs, the resulting ground shaking and shear stress cause semi-dense and loose saturated soils, especially layers of clean sand, silty sand, or even silt; to compact and settle. As the soil compacts, the shifting of particles

within the water-filled voids generates excess pore water pressure. Because earthquakes are usually brief and these soils have low permeability, there isn't enough time for the water to drain. As a result, the excess pore pressure accumulates with each successive loading cycle. Eventually, this pressure becomes so high that the soil particles lose contact with each other, and the effective stress between them drops to zero. When this happens, the soil loses all its shear strength and starts behaving like a viscous liquid with a density equal to the saturated soil's unit weight, leading to significant deformations.

Soils prone to liquefaction are generally categorized as non-cohesive, with those most at risk including clean sands, silty sands with low plasticity, non-plastic silts, and gravels. Cohesive soils, on the other hand, are generally not susceptible to liquefaction. However, if cohesive clay soils are at risk, they must meet all of the following criteria (Das, 2005). If any of these criteria are not met, the cohesive clay soil will not liquefy (Azarafza & Asghari-Kalajahi, 2016):

- The fine particle content (dry weight of soil) smaller than 0.005 mm is less than 15%,
- The liquid limit (LL) is below 35%,
- The soil's moisture content is greater than 0.9 of LL.

Liquefaction can cause a range of damage, including the loss of foundation bearing capacity, soil settlement, compaction of liquefied layers, sand boils, and the ejection of material from buried structures. The most significant impact is often lateral spreading or large deformations. Over the years, several methods have been developed to evaluate liquefaction potential, such as cyclic stress, cyclic strain, energy dissipation, effective stress response, and probabilistic approaches. Among these, the cyclic stress method is the most widely used in seismic geotechnical engineering (Alizadeh, 2016).

In Figure 1, imagine a small soil element in the field under typical conditions. Here, the effective vertical stress on this element is denoted as $\sigma'_v = \sigma_v$, while the effective horizontal stress is $K_0\sigma'_v$, with K_0 being the coefficient of earth pressure at rest. When an earthquake occurs, it induces a cyclic shear stress τ_h on this soil element, as shown in this figure. To study liquefaction in the laboratory, tests must simulate conditions where a constant vertical stress and a cyclic shear stress are applied to a plane within a soil sample. Several laboratory methods have been developed for this purpose, including cyclic triaxial tests, cyclic simple shear tests, and shaking table tests (Day et al., 2012). Among these, the cyclic triaxial test and the cyclic simple shear test are the most commonly used. Laboratory findings reveal that the number of loading cycles needed to trigger liquefaction decreases as the amplitude of shear stress increases and the relative density of the soil decreases. In loose soils, liquefaction can occur with just a few cycles of high shear stress, whereas in dense soils, it may require thousands of cycles with lower stress amplitudes (El Takch et al., 2016). This relationship between soil density, cyclic stress amplitude, and the number of cycles required for liquefaction can be graphically represented by "cyclic resistance ratio (CRR)". These curves are usually normalized by the initial effective overburden pressure to derive the "cyclic stress ratio (CSR)". The definition of CSR may vary depending on the specific test being conducted (Day et al., 2012).

As previously mentioned, the cyclic stress required for liquefaction (resistance) can be calculated by sampling from the site and performing laboratory tests. However, sampling granular soils and transferring them to the laboratory can significantly alter the soil. Additionally, specialized sampling methods such as soil freezing can be used, but these methods are also costly. Therefore, in-situ tests are preferred. These in-situ tests include the Standard Penetration Test (SPT), Cone Penetration Test (CPT), Shear Wave Velocity (V_s), and Becker Penetration Test (BPT). The choice of method depends on various factors such as available equipment, site conditions, costs, and more (Monkul et al., 2015). Table 1 presents the advantages and disadvantages of each method.

The foundation of methods based on in-situ tests relies on extensive empirical data collected from various sites that have been subjected to seismic loading. Some of these sites have experienced liquefaction, while others have not. Identifying liquefaction at these sites is based on observable phenomena, including sand boils, uneven settlements following an earthquake, cracks on the ground surface, and lateral spreading (Seed, 1986). In general, studying the effects of liquefaction on shallow foundations is critical for ensuring the structural integrity of buildings and infrastructure. Liquefaction occurs when saturated soil loses its load-bearing capacity during an earthquake, which can lead to severe damage or even collapse of structures built on shallow foundations. Historical earthquake events have vividly demonstrated how liquefaction can compromise buildings, causing them to sink, tilt, or collapse. Understanding these effects is vital for designing structures that can withstand seismic forces and avoid catastrophic failures (Han, 2015).

The primary concern during any seismic event is the safety of human lives. Investigating how liquefaction affects shallow foundations helps engineers and architects design safer buildings, thereby reducing the risk of fatalities and injuries. A thorough understanding of liquefaction can also enhance emergency preparedness and response planning, allowing for quicker and more effective actions to protect people during disasters. This proactive approach helps in minimizing the potential for loss of life in earthquake-prone areas (Gharbawi et al., 2015). The economic repercussions of liquefaction-induced damage can be substantial, including high repair and reconstruction costs. By studying liquefaction, we can develop strategies to minimize these financial burdens through improved design and construction practices. Additionally, accurate assessments of liquefaction risks influence insurance policies and liabilities, potentially affecting premiums and claims. Understanding the economic implications allows stakeholders to better allocate resources and plan for contingencies (Han, 2015).

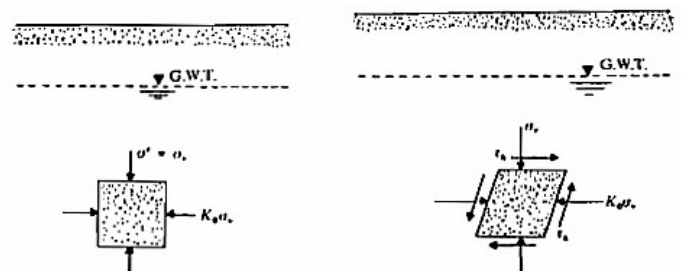


Fig. 1 Stress conditions in soil during liquefaction (Seed, 1986)

Table 1 Advantages and disadvantages of main field-testing methods (Seed & Idriss, 1971)

Parameter	Field-testing methods		
	SPT	CPT	s
Prevalence of test	High	High	Low
Stress-Strain behavior	High	High	Low
Soil layer identification	Poor	Good	Good
Recommended for soil types	Cohesive	Cohesive	All
Sample preparation	Yes	No	No
Engineering properties measurements	Good	Good	Fair

Insights gained from investigating liquefaction effects lead to improved foundation design practices and guidelines. Engineers can implement more resilient design strategies and mitigation measures, such as soil improvement techniques, to enhance the stability of shallow foundations. This knowledge helps in creating buildings that are better equipped to handle seismic forces, contributing to overall structural resilience and safety (Chen, 2015). Studying liquefaction is also essential for updating and enforcing building codes and standards. By incorporating findings into regulatory frameworks, construction practices can be aligned with the latest safety requirements. This ensures that new projects meet established criteria for seismic risk, guiding construction practices and land use planning in earthquake-prone regions. Effective risk assessments based on liquefaction studies help in making informed decisions about where and how to build (Han, 2015). Finally, researching liquefaction contributes to the broader field of geotechnical and earthquake engineering, advancing our understanding of soil behavior under seismic loading. This research fosters innovation in engineering solutions and technologies to address liquefaction issues, improving safety standards and engineering practices. Learning from past events and ongoing studies allows for continuous improvement in construction techniques, ultimately leading to safer and more resilient structures (Bao et al., 2019).

II. LIQUEFACTION ANALYSIS

As stated, there are various methods and procedures used to measure and analyze liquefaction. The following are the main methods that commonly used by geotechnical engineers (Nicholson, 2014):

Laboratory testing: Laboratory testing is fundamental for understanding how soils may behave under seismic stress, especially when assessing liquefaction potential. One of the primary laboratory tests used is the “cyclic triaxial test”. This test involves applying repeated cycles of stress to a soil sample to replicate earthquake conditions and observe how the soil’s shear strength and pore water pressure change. Another crucial method is the “cyclic simple shear test”, which examines the soil’s response to shear stress. These tests are vital for determining how likely a soil is to liquefy and for guiding effective design strategies to mitigate potential risks.

Field testing: Field tests are essential for gathering real-world data on soil behavior. Two widely used methods are SPT and CPT. The SPT involves driving a sample tube into the ground and counting the number of blows required to penetrate the soil. This test helps gauge soil density and strength, which are important for predicting liquefaction potential. The CPT, which

involves pushing a cone-shaped probe into the ground, measures resistance to penetration and pore water pressure directly. This provides detailed information about soil stratigraphy and helps identify layers that might be prone to liquefaction.

In-situ observations: Observing soil behavior in its natural setting, especially during and after earthquakes, provides valuable insights into liquefaction. Techniques like “geophysical surveys” and “piezometer readings” are instrumental in this regard. Geophysical methods such as “seismic refraction” and “ground-penetrating radar” help detect variations in soil properties and identify problematic layers. Piezometers measure changes in pore water pressure in real-time, offering a clear picture of how the soil responds to seismic forces and the likelihood of liquefaction.

Empirical estimations: Empirical methods use historical data and field observations to estimate the potential for liquefaction. The “Seed & Idriss method”, for instance, calculates liquefaction risk based on factors like soil type, earthquake magnitude, and ground shaking intensity. This approach involves computing the CSR and comparing it to CRR derived from laboratory tests. If the CSR is higher than the CRR, there is a higher risk of liquefaction. These empirical methods are particularly useful for quick assessments and initial evaluations in geotechnical investigations.

Numerical simulations: Methods offer a detailed look at how soils might behave during seismic events by modeling soil responses under various conditions. Techniques like FEM and finite difference methods (FDM) are employed to create simulations of soil-structure interactions. These models consider soil properties, loading conditions, and boundary effects to predict soil behavior during earthquakes. Numerical simulations are valuable for testing different scenarios, designing effective mitigation measures, and understanding potential impacts on structures.

Risk evaluation and mitigation: Risk assessment combines data from laboratory tests, field observations, empirical methods, and numerical models to evaluate the impact of liquefaction on structures. This comprehensive analysis helps identify high-risk areas and informs the design of mitigation strategies. Common mitigation techniques include “soil densification”, “grouting”, and improving “drainage conditions”. By assessing risks accurately, construction practices and land-use planning can be adjusted to enhance safety and resilience, ensuring structures can better withstand seismic events.

III. MATERIALS AND METHODS

The growing awareness of issues related to problematic soils and the need for a more detailed understanding of site conditions have led researchers to increasingly rely on computer-based methods for evaluation and expansion of study dimensions (Han, 2015). Among the various approaches, numerical methods have become prominent in geotechnical engineering (Nicholson, 2014). These methods include techniques such as FEM, DEM, Boundary Element Method (BEM), and others (Makia, 2013).

Among these approaches, the FEM stands out for its assumptions and effectiveness in analyzing continuous media such as soil, which is typically considered a continuous and homogeneous medium. The FEM is particularly useful in

geotechnical studies because it allows for detailed analysis of soil behavior under various conditions. When considering soils as elastic-plastic and using the Mohr-Coulomb failure criterion, FEM can effectively model mechanical behavior in response to applied forces and stresses on different site conditions (Han, 2015). PLAXIS is a notable software tool used for this type of analysis. It is a two-dimensional computer program designed for evaluating stability, deformation, settlement, compaction, consolidation, and seepage under both static and dynamic conditions in the field of geotechnics (Byrne et al., 2004). This software allows for a detailed and accurate representation of soil behavior and its interactions with structures, making it a valuable tool for engineers (Makia, 2013).

In this section, we will focus on the modeling and analysis of primary problematic soil phenomena using these advanced numerical methods. By leveraging these techniques, researchers and engineers can better understand the complexities of soil behavior, assess potential risks, and develop more effective mitigation strategies. Overall, the use of numerical methods like FEM offers significant advantages in geotechnical analysis by providing a more precise and comprehensive view of soil-structure interactions. These tools enable the simulation of various scenarios, improving the reliability of predictions and the effectiveness of design solutions. To achieve a precise modeling of the conditions affecting a soil mass, this study aims to address and incorporate most of the critical parameters required for analyzing problematic soils. The modeling process is executed in four stages: geometric modeling of the mass, boundary conditions, property allocation, and definition of behavior models. Each stage is tailored to the specific conditions of the problematic soils being analyzed. A brief overview of the modeling process is provided below.

Geometric modeling: Encountering soils with problematic materials is a common issue in geotechnical engineering. Soils are widely used in the design and construction of various structures. Essentially, structures are built with, in, and on soil. Therefore, dealing with problematic soils is an inevitable aspect of the field. To address this issue comprehensively and analyze the most critical conditions, data regarding problematic soils are collected and analyzed statistically. This involves fitting a normal distribution function to the data and normalizing it to identify the most significant type of problematic soil for the geometric model. In this study, a surface foundation supporting a concrete structure built on problematic soils is used as the basis for the geometric modeling. The geometric models are applied to all types of problematic soils to assess soil-structure interactions and failure conditions under the influence of these soils. Figure 2 illustrates the geometric model prepared for problematic soils.

Boundary conditions: Zienkiewicz et al. (1989) proposed boundary conditions based on fixing the lateral boundaries of meshes and nodes to control the mass body effectively. This approach helps reduce computational errors during analysis, particularly when analyzing consolidation within the model. For dynamic analysis of granular masses, the boundary conditions provided by these researchers are used in this study. The results of applying these boundary conditions are shown in Figure 2. Similar to the geometric model, these boundary conditions are applied across all scenarios of problematic soils. The goal is to

evaluate conditions and interactions affecting the mass and soil-structure interactions for all types of soils.

Property allocation and behavioral models: To determine the behavioral properties and models for the mass body, the material properties are based on a specified range for problematic soils, analyzed individually. This includes the behavior and characteristics of the soils and the resistant concrete surface foundation. Table 2 displays the input parameters for the PLAXIS software used to assign properties to the mass. The behavioral model applied in this study is the Mohr-Coulomb model. This model allows for analysis of failure under normal and shear stresses, accommodating both tensile, compressive and shear failure modes (Tsegaye, 2010).

After completing the geometric modeling, setting boundary conditions, and assigning properties and behavioral models, the model is evaluated under the specified conditions and solved accordingly. The results from this assessment are presented as a mechanical model, which is then used to interpret the prevailing conditions. Mechanical modeling of problematic soils under the foundation is referred to as soil-structure interaction. The outcomes of this mechanical modeling are analyzed to assess the impact and failure mechanisms influenced by different types of problematic soils. Each type of soil is evaluated individually to understand its specific effects and behavior.

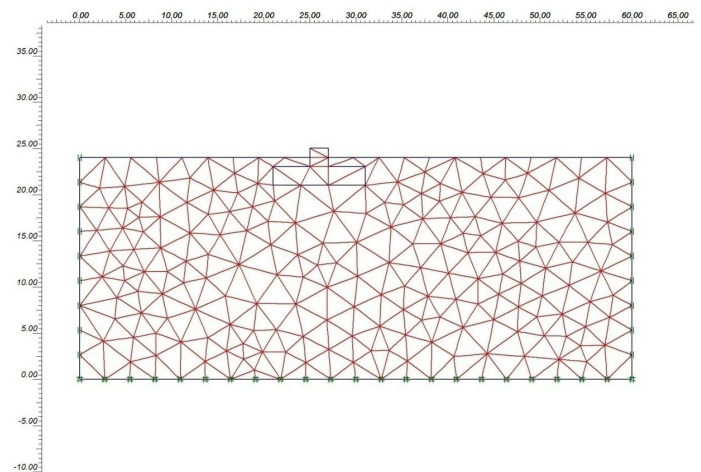


Fig. 2 Square foundation geometrical model with boundary conditions

Table 2 Input geotechnical parameters for ground-foundation

Parameter	Symbol	Unit	Value
Foundation (concrete)	γ_{unsat}	kN/m ³	19.00
	γ_{sat}	kN/m ³	19.00
	E_{ref}	kN/m ³	10000000
	μ	-	0.30
	C_{ref}	kN/m ³	712
	ϕ	Degree	54.9
Liquefiable soil	ψ	Degree	0.00
	γ_{unsat}	kN/m ³	19.00
	γ_{sat}	kN/m ³	21.00
	E_{ref}	kN/m ³	36000
	μ	-	0.33
	C_{ref}	kN/m ³	1.00
	ϕ	Degree	35.0
	ψ	Degree	0.00

IV. RESULTS AND DISCUSSION

The expansion of urbanization and the continuous development of cities have increasingly led to challenges in geotechnical construction due to problematic soils. These issues can either be inherent to the soil or arise from environmental conditions. Among the inherent problematic soils, liquefiable soils are particularly significant, especially in areas prone to seismic activity (Han, 2015). Liquefaction is a critical concern as it can severely compromise the stability of structures built on affected soils. This phenomenon occurs when saturated, loose, granular soils lose their strength and stiffness in response to an applied stress, such as an earthquake, causing the soil to behave like a liquid. This can lead to ground failure and the collapse of buildings and infrastructure (Nicholson, 2014).

In this section of the research, we focus on the numerical modeling of shallow foundations constructed on liquefiable soils. These simulations have been specifically designed to analyze the behavior of these soils under seismic conditions. The results provide insights into the potential risks and help identify the most effective mitigation strategies. Based on the findings from these numerical models, recommendations for soil improvement and stabilization are proposed to enhance the resilience of foundations in areas prone to liquefaction. These strategies aim to reduce the risk of liquefaction and ensure the safety and stability of structures during seismic events.

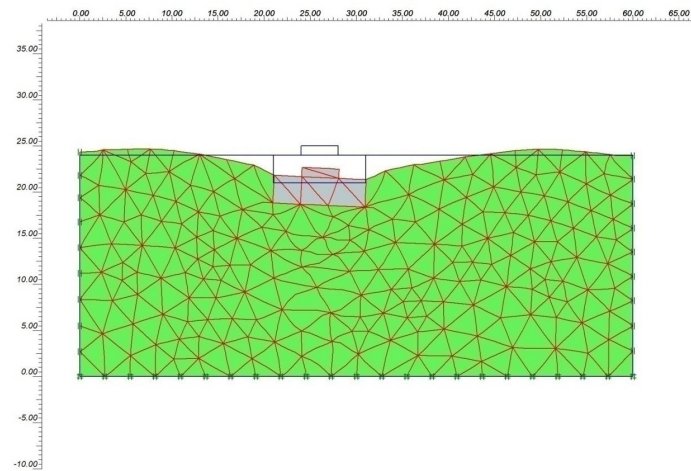


Fig. 3 Deformations from liquefaction in the foundation level

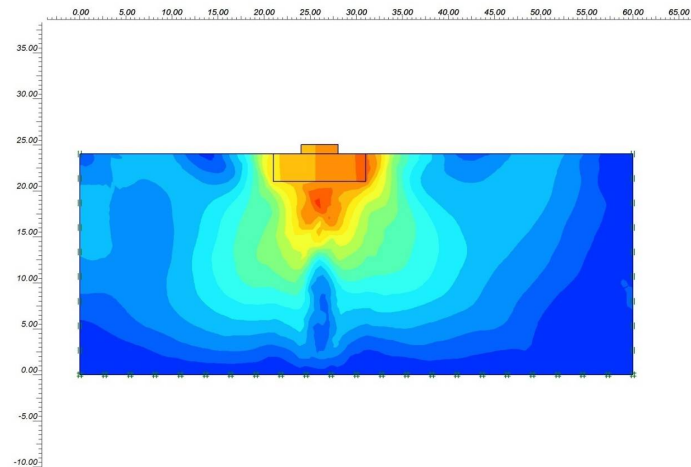


Fig. 4 Displacements from liquefaction in the foundation level

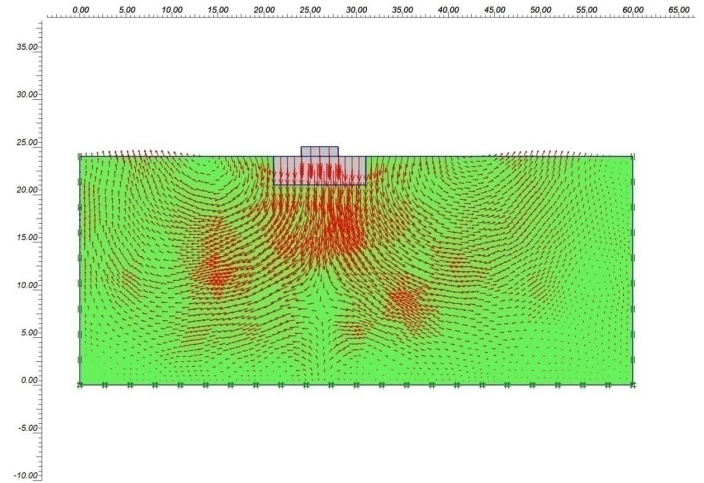


Fig. 5 Strain from liquefaction in the foundation level

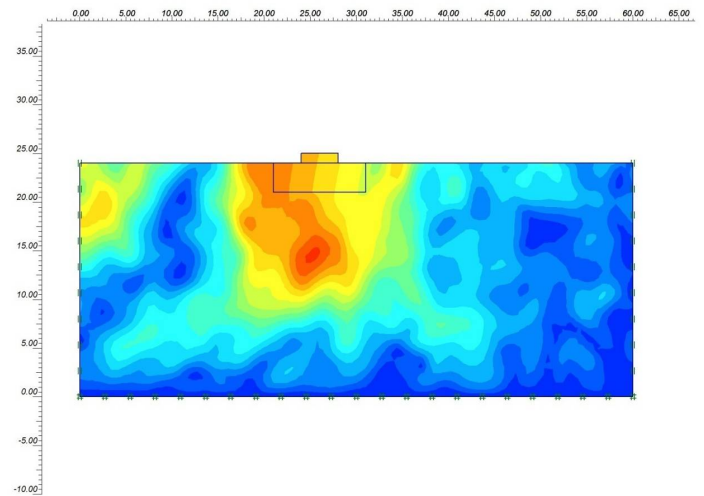


Fig. 6 Shear stress from liquefaction in the foundation level

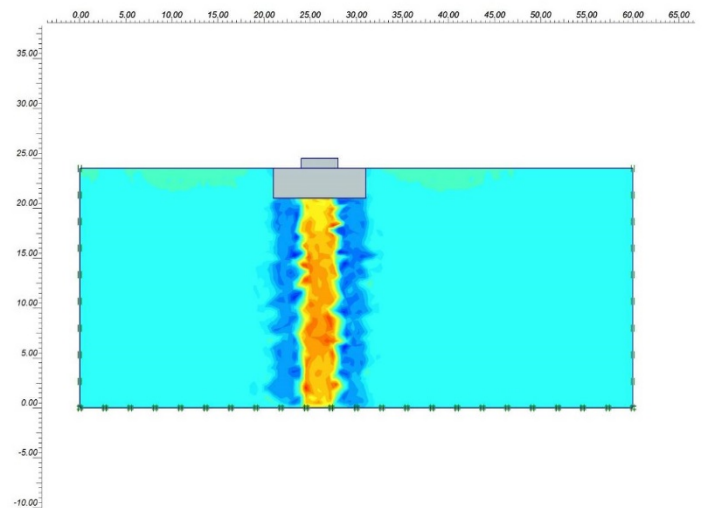


Fig. 7 Stress concentration due to pore water pressure in soil during liquefaction

V. CONCLUSION

Through a detailed numerical study using the finite element method in PLAXIS software, the study has demonstrated that these soils typically experience a reduction in strength and stability under loading, leading to potential foundation failure. The results underscore the importance of understanding the geotechnical behavior of problematic soils, which can significantly compromise the structural integrity of foundations. Furthermore, the research proposes effective improvement and stabilization techniques, emphasizing their necessity in mitigating the adverse effects of soil behavior under shear stress and displacement conditions. So, the findings affirm that numerical modeling with tools like PLAXIS is a reliable approach for assessing soil deformations and ensuring the stability of foundations in challenging soil conditions.

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AUTHORS' CONTRIBUTIONS

Amir Ashrafi conducted the main data analysis, contributed to the data collection, preprocessing, and interpretation, and was responsible for drafting the initial manuscript. Mehdi Kouhdaragh assisted in the development of the methodology and performed validation checks, provided supervision, conceptual guidance, and critical revision of the manuscript. All authors read and approved the final manuscript.

CONFLICT OF INTEREST

The authors have not disclosed any competing interests.

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REFERENCES

Ansari Z., Mansouri-Kia M. (2016). Investigation of the Most Common Methods for Evaluating Liquefaction Potential in Iran. In: *Proceedings of the First National Seminar on Geotechnical Issues of Irrigation and Drainage Networks*, Tehran, Iran.

- Asgari F., Qasemi A. (2009). Liquefaction Hazard Mapping of Lorestan Province. *Civil Engineering Infrastructures*, 43(1), 95-103. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2006\)132:6\(705\)](https://doi.org/10.1061/(ASCE)1090-0241(2006)132:6(705)).
- Azarafza M., Asghari-Kaljahi E. (2016). *Applied Geotechnical Engineering*. Negarkhane Publication, Isfahan, Iran.
- Bao X., Jin Z., Cui H., Chen X., Xie X. (2019). Soil liquefaction mitigation in geotechnical engineering: An overview of recently developed methods. *Soil Dynamics and Earthquake Engineering*, 120, 273-291. <https://doi.org/10.1016/j.soildyn.2019.01.020>.
- Byrne P.M., Park S.S., Beaty M., Sharp M., Gonzales L., Abdoun T. (2004). Numerical modelling of dynamic centrifuge tests. In: *Proceedings of the 13th World Conference in Earthquake Engineering*, British Columbia, Canada.
- Chen G. (2015). *Liquefaction of sands*. Ph.D. Dissertation, Harvard University, Cambridge, UK.
- Das B.M. (2005). *Principles of Geotechnical Engineering (6th Edition)*. CL Engineering press, London, UK.
- Day S.H., Seed H.B., Idriss I.M. (2012). Analysis of Soil Liquefaction: Niigata Earthquake. *ASCE Journal of Soil Mechanics & Foundations Division*, 93(SM3), 83-108. <https://doi.org/10.1061/JSFEAQ.0000981>.
- El Takch A., Sadrekarimi A., El Naggat H. (2016). Cyclic resistance and liquefaction behavior of silt and sandy silt soils. *Soil Dynamics and Earthquake Engineering*, 83, 98-109. <https://doi.org/10.1016/j.soildyn.2016.01.004>.
- Gharbawi T., El-Mohammad Y., Masayuki T. (2015). Estimating deformation due to soil liquefaction in Urayasu city, Japan using permanent scatterers. *ISPRS Journal of Photogrammetry and Remote Sensing*, 109, 152-164. <https://doi.org/10.1016/j.isprsjprs.2015.09.002>.
- Han J. (2015). *Principles and Practice of Ground Improvement (1st Edition)*. Wiley, New Jersey, USA.
- Makia A. (2013). *Evaluation of the UBC3D-PLM constitutive model for prediction of earthquake induced liquefaction on embankment dams*. MSc Thesis, Delft University of Technology (TU Delft), Netherlands.
- Monkul M., Murat M. (2015). Estimation of liquefaction potential from dry and saturated sandy soils under drained constant volume cyclic simple shear loading. *Soil Dynamics and Earthquake Engineering*, 75, 27-36. <https://doi.org/10.1016/j.soildyn.2015.03.019>.
- Nicholson P.G. (2014). *Soil Improvement and Ground Modification Methods (1st Edition)*. Butterworth-Heinemann press, Oxford, UK.
- Seed H.B. (1986). Use of SPT and CPT tests for evaluating the liquefaction resistance of sands. *ASCE Use of In-Situ Tests in Geotechnical Engineering*, 6, 281-302.
- Seed H.B., Idriss I.M. (1971). Simplified procedures for evaluating soil liquefaction potential. *ASCE Journal of Soil Mechanics and Foundations Division*, 97(SM9), 1249-1273.
- Tsegaye A. (2010). *Plaxis liquefaction model*. External report, PLAXIS knowledge base: www.plaxis.nl
- Verudgo R. (2005). *Characterization of sandy soil behaviour under large deformation*. Ph.D. Thesis, University of Tokyo, Bunkyo, Japan.
- Yuan X., Cao Z. (2011). A fundamental procedure and calculation formula for evaluating gravel liquefaction. *Earthquake Engineering and Engineering Vibration*, 10(3), 339-347. <https://doi.org/10.1007/s11803-011-0070-4>.
- Zhang Y.G., Qiu J., Zhang Y., Wei Y. (2021). The adoption of ELM to the prediction of soil liquefaction based on CPT. *Natural Hazards*, 107(1), 539-549. <https://doi.org/10.1007/s11069-021-04594-z>.
- Zienkiewicz O.C., Bicanic N., Shen F.Q. (1989). Earthquake input definition and the transmitting boundary conditions. *Proceedings Advances in Computational Nonlinear Mechanics I*, Springer-Verlag, pp. 109-138.